HAO SHABAR Instrument and Observations

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Summary: We have designed and constructed a SHABAR array and deployed it to Mauna Loa Solar Observatory to measure the height variation of the atmospheric structure constant. This allows us to infer the height dependence of the seeing which will determine the required height of the COSMO telescopes. This report describes the design and construction of the scintillometer array, the data analysis procedure and the results of the measurements taken at Mauna Loa with implications for the design of the COSMO Large Coronagraph. We find that the entrance pupil of the COSMO Large Coronagraph should be more than 14.7 meters above the ground to achieve a median seeing of 2 arcseconds at 530 nm.

Introduction

Seykora found that the scintillation of the solar intensity signal correlates with solar image quality (Seykora, 1993). This was justified by Beckers in terms of atmospheric optics (Beckers, 1993). Significantly, Beckers realized that an array of scintillometers with varying distances between them could be used to infer the height variation of the atmospheric structure constant, $C_n^2$, which quantifies the refractive index variations of the atmosphere responsible for seeing. He called this technique SHAdow Band Ranging (SHABAR) and developed this technique in a series of papers (Beckers et al., 1997; 1998; Beckers 1999; 2001). SHABAR arrays were used in the ATST site survey (Hill, et al., 2004) to infer the $C_n^2$ profiles at a variety of sites in order to set the required height of the telescope. SHABAR arrays have also been installed in Tenerife and LaPalma (Sliepen et al., 2009).

We have designed and constructed a SHABAR array and deployed it to the Mauna Loa Solar Observatory (MLSO) to measure the height variation of the seeing to set the height requirement for the COSMO Large Coronagraph and suite of instruments. The height of the telescope enclosure is a major cost driver for any telescope. This report describes the design and construction of the scintillometer array, the data analysis procedure and the results of the measurements. We also discuss the implications of the measurements for the design of the COSMO Large Coronagraph.
SHABAR Theory

The theory behind the SHABAR has been developed extensively in the series of papers by Beckers and co-authors (Beckers et al., 1997; 1998; Beckers 1999; 2001) and will be described only briefly here. The scintillation measured in a detector pointed at the Sun is due to the refractive index variations in the cone of light extending from the detector to the solar disk. The basic concept behind the SHABAR is that the scintillation signal from two detectors will begin to be correlated at the height at which their light cones begin to overlap. This height depends primarily on the separation of the detectors. An array of scintillometers with different separations will have light cones that overlap at a range of heights above the detectors. The height dependence of the refractive index variations responsible for the scintillation can be determined by the co-variances of the signals from the detectors comprising the scintillometer array. This is illustrated in Figure 1 which is taken from Hill, Radick and Collados (2003).

\[ B_I = \int_{h=0}^{\infty} C_n^2(h)K(h,r)dh , \]  

Following Beckers (2001), the expression for the measured covariances of the normalized intensity fluctuations from two detectors is given by
where $C_n^2$ is the atmospheric structure constant which describes the turbulence of the atmosphere, and $K$ are the sensitivity kernels over which the turbulence is averaged. The kernels are given by

$$K(h,r) = \frac{0.38 \times 32 \pi h^2 \sec^3 z}{(a + ah \sec z)^{7/3}} Q\left(\frac{r}{a + ah \sec z}\right),$$

(2)

where $h$ is the height above the detector, $z$ is the zenith angle, $a$ is the detector diameter, $r$ is the detector pair separation and $\alpha = 2 \tan(\theta/2)$ with $\theta$ being the solar angular diameter. $\theta$ can usually be approximated by its average value of 1920 arcseconds, which gives a value of $\alpha = 0.0093$. The function $Q$ is given by

$$Q(s) = \int_{f=0}^{\infty} J_1(\pi f)^2 J_0(2\pi fs) f^{-2/3} df.$$  

(3)

In practice, $Q$ is expensive to compute, so it is tabulated and interpolated when needed. $Q(s)$ is shown in Figure 2. Note that $Q(0) = 0.450$ and $Q$ crosses 0 at a value of $s = 0.862$.

In general, the intensity fluctuations are measured with an array of $N$ photometers with $N$ typically around 6. This provides $N(N-1)/2$ detector pairs. The typical amplitude of the intensity fluctuations is $10^{-3}$. The cross-covariances of these fluctuations are computed and used to infer the structure constant $C_n^2$ as a function of height through the inversion of equations 1-3. In addition, any detector can provide a zero separation ($r = 0$) autocovariance which also provides useful information. Then the total number of detector separations will be $N(N-1)/2 + 1$. 

![Figure 2. The function Q(s)](image)
The HAO SHABAR

The design of the HAO SHABAR instrument follows closely from the work of Beckers (2001) and from the work of the Utrecht group (Sliepen et al., 2009). It consists of an array of six scintillometers comprised of photodiodes placed behind mini-telescopes. To reduce design time and cost, we constructed the HAO SHABAR telescopes with off-the-shelf components, obtained primarily from ThorLabs.

Figure 3. Schematic of HAO SHABAR telescope. Sunlight enters from top. All components are (1/2”) 12.7 mm diameter. An image of the aperture stop is formed 4.0 mm below the bottom of the lens tube on the photodiode. All optics are mounted in a single SM05M30 lens tube. Red arrows highlight assembly dimensions.
An optical schematic is shown in Figure 3. To reduce the sensitivity to pointing errors, the telescopes form an image of the entrance aperture on the photodiode. Each telescope is constructed from ½-inch ThorLabs lens tubes. Light is collected by a ½-inch objective lens that is stopped down to an aperture of 2 mm. A field stop of 3.17 mm is located in the focal plane followed by a Fabry lens which forms an image of the entrance aperture on a ThorLabs PDA36A photodiode. The BG18 filter along with the responsivity of the silicon detector limit the optical bandwidth to be 150 nm centered at 495 nm. Short wavelengths are preferred due to the increased scintillation signal at shorter wavelengths. A list of components is given in Appendix A, while Appendix B gives instructions for the assembly of the SHABAR telescopes.

The telescopes are mounted through drilled holes in an aluminum extrusion and held by plastic screws. The positions of the telescopes for the HAO SHABAR array are: 0.0, 24.2, 55.0, 95.6, 211.5 and 381.7 mm. These six scintillometers can be combined to form 15 unique separations, and with the zero separation case yield a total of 16 separations. Data is taken sequentially from the six detectors at a rate of 240 kHz. This is then averaged down by a factor of 40 to give an effective sampling rate of 1 kHz for each scintillometer. Data is taken continuously in 10 second bursts and written to disk. The HAO SHABAR was tested in Boulder in November 2012 and deployed to the MLSO spar in December 2012. A picture of the SHABAR mounted on the front of the MLSO spar is shown in Figure 4.

Figure 4. Picture of the HAO SHABAR array mounted to the front of the MLSO spar.
Inversion of the Data

Figure 5 shows the averaging kernels for the 16 detector separations computed with Equations 2 and 3 assuming observation at the zenith and a solar radius of 960 arcseconds. The top plot shows the kernels on a linear scale and the bottom plot on a logarithmic scale. The leftmost line is the zero separation case and the lines increase to the right with increasing separation. The height at which the non-zero separation kernels turn over can be estimated from Q(s) being zero at s = 0.862. Setting the argument of Q in Equation 2 equal to 0.862 (assuming z = 0), gives

\[
h = 124.7 \, r - 0.512
\]

for the HAO system with \( h \) and \( r \) in meters. This shows the height below which a detector pair loses sensitivity is linearly related to the detector separation.

Figure 5 illustrates the basic properties and limitations of the SHABAR data. The HAO system provides excellent height information on \( C_n^2 \) from close to the telescope out to about 500 m where the kernels converge. Above this height, the kernels become degenerate and the measurements no longer contain any information on the height dependence of \( C_n^2 \). The slow decrease of the kernel amplitude for large \( h \), as \( h^{-1/3} \), means that turbulence will still contribute to the scintillation to some degree at high altitudes. The zero separation fluctuations (auto-covariances) are dominated by the near telescope turbulence and have an area under the curve much greater than the non-zero separations. This means that the auto-covariance of a given detector will typically be much larger than the cross-covariance with the other detectors.

The dotted vertical lines on the top plot of Figure 5 indicate the heights at which the inversions were computed. The height grid was chosen to increase linearly with the \( \log_{10} \) of height. Importantly, the inversions were found to be much more stable if only one height bin was used in the low and high altitude regions where the kernels lose height discrimination. The top of the highest height bin was chosen to be 5 km.

Following the ATST and Utrecht groups, the inversion of \( C_n^2 \) was accomplished by using the Nelder-Mead (amoeba) algorithm to determine the values of \( C_n^2 \) at the heights shown in Figure 5 that minimize the difference between the observed and modeled covariances computed with Equations 1-3 according to

\[
E_m = \frac{1}{N_r} \sum_{i=1}^{N_r} \left[ B_{\text{model}}(r_i) - B_{\text{obs}}(r_i) \right]^2 ,
\]

where \( N_r \) is the number of separations and the B are the covariances.
Figure 5. The scintillation Kernels for the HAO SHABAR array are the solid curves shown on a linear scale (top plot), and on a logarithmic scale (bottom plot). The zero separation curve is to the left and the curves move to the right with increasing detector separation. The case shown is for observations at the zenith assuming a solar radius of 960 arcseconds. The dotted vertical lines on the top plot indicate the heights at which the inversions were computed.

In addition, a smoothing term was included in the merit function to insure reasonably smooth $C_n^2$ profiles, given by

$$E_s = \frac{1}{2(N_h - 1)} \sum_{i=2}^{N_h} [\log C_n^2(h_i) - \log C_n^2(h_{i-1})]^2,$$

where $N_h$ is the number of height bins. The quantity which is minimized is
\[ E = E_m + \Lambda E_s , \]  

where \( \Lambda \) is the regularization coefficient that sets the degree of smoothness imposed on the solution, which has been set to \( 10^{-15} \).

**Results of the Inversion**

Data were obtained with the HAO SHABAR array for several days in Boulder for testing purposes. Following testing, the array was deployed onto the solar spar at MLSO and has been used to obtain observations regularly. Sample \( C_n^2 \) profiles are shown in Figures 6 and 7. The profiles are hourly averages. Figure 6 shows the profile obtained in the parking lot of the Boulder Center Green Lab taken on 9 November 2012, while Figure 7 shows the results for a typical day obtained at MLSO, 11 February 2013. The primary difference between the data taken at the two locations is that the data taken at MLSO typically show a significant contribution to the structure constant within the first few meters above the SHABAR instrument. This is presumably due to seeing generated within the dome or immediately outside it. Early in the morning at MLSO, the seeing several meters above the SHABAR is excellent with typical values below one arcsecond. During the day the meteorological conditions change to an upslope condition which is characterized by poorer seeing conditions, around two arcseconds or so. The transition to upslope conditions can be very abrupt or gradual.

Once the values of \( C_n^2 \) have been obtained from the inversion, they can be converted into seeing by first computing the value of the Fried parameter through the integral over height of \( C_n^2 \) (e.g. Beckers, 2001).

\[ r_0 = 0.18466 \lambda^{6/5} \cos^{3/5} z \left[ \int_{h=H}^{\infty} C_n^2(h)dh \right]^{-3/5} , \]

where \( \lambda \) is the wavelength (here 495 nm), \( z \) is the angle from zenith, and \( r_0 \) is Fried’s parameter which quantifies the coherence size of turbulence elements. Values of \( r_0 \) for various heights above the SHABAR can be obtained by taking the integral in Equation 8 from the desired height, \( H \), to infinity.

The full-width-half-maximum (fwhm) seeing is computed from Fried’s parameter following Dierickx (1992),

\[ \epsilon_{fwhm} = 0.98 \frac{\lambda}{r_0} . \]

The integral in Equation 8 needs to be taken over the entire atmosphere. As discussed above, due to the limitations of the averaging kernels, these SHABAR measurements provide the \( C_n^2 \) profile only up to 5 km. This means that the \( r_0 \) values obtained will not include any high altitude (\( h > 5 \) km) seeing component and the seeing will be slightly
underestimated \((r_0\) overestimated). To account for the high altitude seeing in a statistical way, we note that the total seeing is given by the sum of the seeing at different layers in the atmosphere according to the expression (Pant, Stalin and Sagar, 1999)

\[
\epsilon_{fwhm} \text{(total)} = \left( \sum_{i} \epsilon_i^{5/3} \right)^{3/5}.
\]  

(10)

We can then obtain an estimate of the total seeing above MLSO by adding, according to Equation 10, the seeing estimate obtained by the SHABAR with an estimate of the seeing from 5 km to infinity.

We have estimated the high altitude seeing above MLSO in two different ways. In the first way, we noted that when upslope conditions exist through most of the day, the \(C_n^2\) profiles converge to the pattern shown in top panel of Figure 7 starting at 10 am. There, the structure constant shows a power law dependence with height (excluding the lowest heights), varying approximately as \(h^{-4/3}\) as predicted by Wyngaard (1971) for seeing near the ground. We have performed a power law fit of hourly averages with height and find that the median exponent for all of the data is -1.2 with a median proportionality constant of \(10^{12.2}\). Then, we find that

\[
C_n^2(h) \approx 10^{-12.2} h^{-1.2}.
\]  

(11)

Using this height dependence of the structure constant, we can estimate the high altitude seeing by substituting it into Equation 8 and performing the integral. We set the limits of integration to be from 5 km with an upper limit of 30 km. This results in a value of \(r_0 = 23\) cm which corresponds to a seeing of 0.44 arcseconds.

A second estimate to the high altitude seeing above Mauna Loa was obtained using the parameterizations of Racine (2005). He provides estimates of the various components of seeing with altitude above Mauna Kea and other sites. Evaluation of Racine’s parameters imply that the free atmosphere would contribute about 0.28 arcseconds of seeing for a hypothetical observer at an altitude of 8400 m above the island of Hawaii (5 km + MLSO elevation of 3400 m).

These two estimates for the high altitude seeing are reasonable and approximately consistent. To be conservative, we use the estimate of 0.44 arcseconds for the high altitude seeing above MLSO. We then obtain estimates of the total seeing by using the expression

\[
\epsilon_{fwhm} \text{(total)} = \left( \epsilon_{SHABAR}^{5/3} + 0.44^{5/3} \right)^{3/5},
\]  

(12)

where \(\epsilon_{SHABAR}\) is the seeing obtained from inverting the SHABAR data. Note that since \(5/3\) is approximately equal to 2, the seeing contributions from the two layers add almost quadratically. This makes the contribution from high altitudes nearly negligible except for the best seeing conditions, and the results of this study insensitive to the value of the high altitude seeing chosen.
Figure 6. Top Panel: Hourly average of the $C_n^2$ profile obtained with the HAO SHABAR in the Center Green parking lot in Boulder on 9 November 2012. Middle Panel: Estimate of the Fried parameter with height. Lower Panel: Corresponding estimate of the seeing at a height of 7.3 meters above the SHABAR.
Figure 7. Top Panel: Hourly averages of the $C_n^2$ profiles obtained with the HAO SHABAR on the spar at MLSO on 11 February 2013. Note the strong turbulent layer in the first few meters above the SHABAR instrument. Middle Panel: Estimate of the Fried parameter with height. Bottom Panel: Corresponding estimate of the seeing at a height of 7.3 m above SHABAR.
Conclusions

Figure 8 is the result of the analysis of nine months of data taken at MLSO starting in December 2012. It shows the height dependence of the median seeing for wavelengths of 530, 656 and 1083 nm. The height above the ground was calculated by adding 7 m to the heights above the SHABAR. This 7 m corresponds to the height of the SHABAR on the MLSO spar when the spar is pointed to the vertical. The corresponding variation of the Fried parameter, $r_0$, with height can be found using Equation 9. The resulting variation of both the mean and median variation of $r_0$ is shown in Figure 9. It is plotted using the same axes as the corresponding plots found by the ATST project for Haleakala (Figure 10.1 of the ATST Site Report, Hill et al., 2004) for comparison. The regime of “dome” seeing has been removed from both Figures 8 and 9. The height dependence of the median values of $r_0$ for Mauna Loa from this study, and Haleakala from the ATST study are remarkably similar. They both show an increase in $r_0$ from about 4 cm at a height of 10 m, to a value of 7 cm at a height of 80 m. The variation of the mean values of $r_0$ differ significantly between Mauna Loa and Haleakala. The mean values for Mauna Loa are very similar to the median values, as is shown in Figure 9. However, the mean values of $r_0$ found by ATST for Haleakala are much larger than the median values which implies a relatively small number of measurements with extremely large values of $r_0$. Our conclusions will be based on the median values of the seeing and $r_0$.

It is noteworthy that the seeing, and the corresponding $r_0$, show a relatively weak dependence with height for both Mauna Loa and Haleakala.

Given that the cost of a telescope will be a monotonically increasing function of the size of the telescope enclosure, this pushes any seeing vs. cost trade study towards lower telescope heights. The seeing requirement for the COSMO Large Coronagraph is for a median fwhm seeing of 2 arcseconds at a wavelength of 530 nm. This requirement, along with the height dependence of the seeing directly determines the height at which the entrance aperture of the COSMO LC must be placed above the ground.

How high does the COSMO LC need to be? Interpolation of the seeing at 530 nm from Figure 8 indicates that the LC seeing requirement will be met at a height of 13.1 m. Assuming that the LC will be used to observe the corona through no more than 4 airmasses, one can compute the mean elevation of the telescope over the course of the year will be 42.8 degrees. If the LC is 10 m in length, then the average height of the top of the telescope will be 1.6 m below is height when the telescope is vertical. So, the height of the COSMO LC lens, when pointed at the zenith, needs to be at a height of 13.1 + 1.6 = 14.7 m.

The analysis indicates that the seeing requirement for COSMO can be met if the entrance aperture of the LC will be placed greater than 14.7 meters above the ground. Since the ChroMag instrument has a seeing requirement of 2 arcseconds at 589 nm, the other instruments in the COSMO suite need to be placed at a similar height above the ground.
Figure 8. Height dependence of the median FWHM seeing at MLSO. Blue, Green and Red curves correspond to wavelengths of 530, 656 and 1083 nm respectively. The dome seeing has been removed.

Figure 9. Variation of the Fried parameter with height for Mauna Loa. The blue curve is the median and the red curve is the mean value.
References


Appendix A

Component List for HAO SHABAR

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<th>Quantity</th>
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Appendix B

HAO SHABAR Telescope Assembly Instructions

Please refer to the SHABAR Optical System Schematic. The SHABAR optical system has three groupings of optical elements. They will be assembled from bottom to top.

1) Place one SM05 retaining ring 28.55 mm from the bottom end of the SM05M30 lens tube. Insert LA1540 lens into bottom of lens tube, curved side towards bottom. Secure with second retaining ring from bottom.

2) From the top end of the lens tube, place a retaining ring 36.66 mm from the top of the lens tube. Insert field stop from front end of lens tubes, black side towards front. Secure from front with retaining ring.

3) From top of lens tube, insert retaining ring 20.73 mm from top of lens tube. Insert BG18 filter, aperture stop with black side towards front, and objective lens, flat side down. Secure from front with retaining ring.

Note: The lens tube has 40 threads per inch which corresponds to 0.635 mm per revolution of the retaining ring.