Measurement Errors for Coronal Magnetic Field Parameters

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Summary: This note calculates the measurement errors in the line-of-sight (LOS) magnetic field strength, the LOS velocity, and the plane-of-sky (POS) direction of the magnetic field from observations of a coronal emission line in the presence of background light. The first part derives expressions for the errors in the parameters as a function of the number of detected photons for a coronal emission line, assuming photon noise. The second part presents a flux budget for the coronal observations assuming the FeXIII line at 1074.7nm. The note concludes with plots of the expected errors in the magnetic field strength, line-of-sight velocity and plane-of-sky magnetic field direction, as a function of the coronal intensity, the background intensity, and the coronagraph aperture.

Introduction
Measuring magnetic fields in the solar corona is difficult. This is due to the fact that the magnetic fields there are weak and the coronal emission lines are broadened by the million degree plasma and by non-thermal effects. The science drivers for the COSMO Large Coronagraph (LC) are presented in the Science Requirements Document (Landi et al., 2015; COSMOLC-RQ-1000). This technical note derives expected errors on the coronal magnetic field parameters to be obtained with the COSMO LC in order to insure that the instrument will meet requirements. The requirements that this technical note will address are:

- The LC must observe the LOS magnetic field with a sensitivity of 1 G at a spatial resolution of 2 arcseconds in 15 minutes for coronal intensity of 10 ppm using the FeXIII 1074.7 nm line.
- The LC must observe the LOS Doppler velocity with a sensitivity of 30 m/s in 30 seconds.
To meet these and the other science requirements, the LC will have the following properties that are relevant to this discussion:

- 1.5 meter aperture
- 1 degree field-of-view
- 2 arcseconds spatial resolution
- Filtergraph with 0.14 nm FWHM spectral resolution at 1074.7 nm
- Filtergraph throughput of 0.094 at 1074.7 nm

**Expected Magnetic Field Signal**

As background for the discussion, we compute the magnitude of the expected magnetic signal. The LOS magnetic field will be observed through the Zeeman effect in which the magnetic field signature is present in the variation of the circular polarization, Stokes V signal, with wavelength across the line profile. For the small magnetic fields present in the corona and the broad emission lines used to observe them, the Zeeman shift is a small fraction of the line width and the so-called weak field approximation is valid. The Stokes V signal in the weak field approximation is (e.g. Landi Degl’Innocenti, 1992)

$$V = -kB_{\text{LOS}} \frac{\partial I}{\partial \lambda}. \quad (1)$$

The value of $k$ is $4.67 \cdot 10^{-12} \, g \, \lambda^2 \, \text{nm/G}$ where $g$ is the Lande-g value, $\lambda$ is the wavelength in nm and $B$ is the magnetic field in G.

Assuming the emission line is Gaussian, its profile is given by

$$I = I_0 e^{-\frac{(\lambda - \lambda_0)^2}{\Gamma^2}}. \quad (2)$$

where $I_0$, $\lambda_0$, and $\Gamma$ correspond to the central intensity, central wavelength, and e-folding half-width of the emission line, respectively. It can be shown that the intensity derivative has its minimum and maximum at $|\lambda - \lambda_0| = \Gamma / \sqrt{2}$ with the maximum value of the derivative

$$\frac{\partial I}{\partial \lambda} (\text{max}) = \frac{\sqrt{2} I_0}{\sqrt{e} \, \Gamma}. \quad (3)$$

Since Stokes V will have a minimum value of equal and opposite sign as the maximum, the Stokes V amplitude divided by the intensity will be

$$\frac{V}{I_0} (\text{amplitude}) = kB_{\text{LOS}} \frac{2\sqrt{2}}{\sqrt{e} \, \Gamma}. \quad (4)$$
We can estimate the value of the e-folding half width, $\Gamma$, for the FeXIII line at 1074.7 nm by assuming thermal Doppler broadening at a temperature of 1.8 MK, which results in $\Gamma = 0.0829$ nm. Adding a non-thermal line width of 20 km/s (0.0716 nm) in quadrature gives a final e-folding line width of 0.1095 nm (30.8 km/s). Also, $g = 1.5$ for the FeXIII 1074.7 nm line resulting in $k = 8.1 \cdot 10^{-6}$ nm/G and

$$\frac{V}{I_0} (amplitude) = 1.3 \cdot 10^{-4} B_{LOS}(G).$$  \hspace{1cm} (5)

This drives the COSMO LC requirement to measure the Stokes V amplitude to a level of $\sim 10^{-4}$ to reach 1 G sensitivity.

**Mathematical Formulation**

In this section, we will develop a mathematical formulation by which we can derive analytic expressions for the errors on the properties of the corona inferred from measurements of the Stokes profiles of an emission line.

The general problem is to determine the parameters of a profile of a coronal emission line given a set of observations of the intensity at various wavelengths across the line. Assuming a model for the line profile which is a function of some number of parameters, we can determine the parameters that best fit the observations and relate these parameters to the properties of the coronal magnetic field. Here we are interested in the uncertainty in the fit parameters and the resulting uncertainties in the magnetic field parameters.

In general, the best estimate of a parameter, $a$, is given by the standard equation (e.g. Taylor, 1982)

$$a_{best} = \frac{\sum a_i W_i}{\sum W_i},$$ \hspace{1cm} (6)

where the subscript, $i$, denotes the individual measurements of the parameter, and the $W_i$ are weights, given by

$$W_i = \frac{1}{\sigma_{a_i}^2}.$$ \hspace{1cm} (7)

The uncertainty in the best estimate of the parameter is

$$\sigma_{a_{best}}^2 = \frac{1}{\sum W_i}.$$ \hspace{1cm} (8)

Given a model for the intensity profile of the spectral line, the variance of the parameter values can be computed from the partial derivative of the intensity profile with respect to each parameter. The variance in the individual measurement of the parameter, $a_i$, is given
by standard propagation of errors, neglecting any correlations between the uncertainties in the parameters, as

\[ \sigma_{a_i}^2 = \frac{\sigma_{I_i}^2}{\left( \frac{\partial I}{\partial a_i} \right)^2} , \]  

(9)

where \( I_i \) is the intensity profile observed at \( i \) points and \( \sigma_{I_i}^2 \) are the variances of the intensity measurements. Intensity is in units of photons per unit wavelength. Combining equations 7, 8, and 9 gives

\[ \sigma_{a_{best}}^2 = \left[ \sum \frac{\left( \frac{\partial I}{\partial a_i} \right)^2}{\sigma_{I_i}^2} \right]^{-1} . \]  

(10)

The sum is assumed to be taken over the entire line profile.

For the coronal emission lines considered here, the intensity of an emission line profile is well approximated by a Gaussian as in Equation 2. The intensity is in units of photons \( / \text{cm}^2 / \text{nm} / \text{arcsecond}^2 / \text{second} \) or equivalent. For the moment, we consider only the photon and wavelength dependence of the intensity. The collecting area, spatial resolution and time dependence will be included later.

Requiring that the integral of the line profile over wavelength be equal to the total number of photons in the emission line, \( N \), will ensure proper normalization of the intensity into units of photons per unit wavelength. Then

\[ \int_{-\infty}^{\infty} I_0 e^{-\frac{(\lambda - \lambda_0)^2}{\Gamma^2}} \, d\lambda = N , \]  

(11)

and \( I_0 = \frac{N}{\Gamma \sqrt{\pi}} . \)  

(12)

Historically, it has been customary to define the line center intensity, \( I_0 \), as a fraction of the intensity of the center of the solar disk at that wavelength.

The uncertainty in the value of \( \lambda_0 \) can be determined using equation 10. The partial derivative of Equation 2 with respect to wavelength is

\[ \frac{\partial I}{\partial \lambda_0} = -2 \frac{\lambda - \lambda_0}{\Gamma^2} . \]  

(13)

Next, we need an estimate of the uncertainty of the intensity measurements. For the purposes of this note, we will assume that the uncertainties are limited by photon noise.
Other noise sources can be included in a straightforward manner. The number of photons \( n_i \) in a wavelength sampling interval \( \Delta \lambda \), is given by \( n_i = I_i \Delta \lambda \). Then it follows that \( \sigma_{n_i} = \sigma_i \Delta \lambda \). Assuming photon noise, \( \sigma_{n_i}^2 = n_i \), we obtain

\[
\sigma_{I_i}^2 = \frac{I_i}{\Delta \lambda}.
\]  

(14)

Substituting Equations 13 and 14 into equation 10 gives

\[
\sigma_{\lambda_0}^2 = \frac{1}{\sum 4 \left( \frac{\lambda - \lambda_0}{\Gamma} \right)^2 \Delta \lambda}.
\]  

(15)

For a well sampled line profile, the sum can be approximated by the integral

\[
\sigma_{\lambda_0}^2 = \frac{1}{\int_{-\infty}^{\infty} 4 \left( \frac{\lambda - \lambda_0}{\Gamma} \right)^2 \Delta \lambda}.
\]  

(16)

Substituting \( I \) from Equation 2, using the normalization of \( I_0 \) from equation 12 and evaluating the integral gives

\[
\sigma_{\lambda_0} = \frac{\Gamma}{\varepsilon_I \sqrt{2N}}.
\]  

(17)

We have included the Stokes I modulation efficiency, \( \varepsilon_I \), in the equation to properly capture the penalty on the noise from the measurement of Stokes I from which the central wavelength was computed (del Toro Iniesta and Collados, 2000). The uncertainty in the Doppler velocity, \( \nu \), can be found by multiplying Equation 17 by \( c/\lambda \),

\[
\sigma_\nu = \frac{c \Gamma}{\lambda \varepsilon_I \sqrt{2N}}.
\]  

(18)

This expression is identical to the one (save for the modulation efficiency factor) presented in Penn et al. (2004) for the case of a triangular emission line profile.

The magnetic field strength can be determined through the wavelength shift induced by the Zeeman effect and Equation 17. Then the magnetic field uncertainty will be given by

\[
\sigma_B = \frac{\Gamma}{S \varepsilon_V \sqrt{2N}}.
\]  

(19)

\( S \) is the Zeeman sensitivity in units of wavelength shift per magnetic field strength (e.g. nm/G), and \( \varepsilon_V \) is the Stokes V modulation efficiency. For a normal triplet, the Zeeman splitting is \( \Delta \lambda = 4.67 \times 10^{12} \, g \, \lambda^2 \) (nm/G) (Lani Degl’Innocenti, 1992) with \( \lambda \) in nm. Then
\[
\sigma_B = \frac{\Gamma}{6.60 \cdot 10^{-12} \epsilon \lambda^2 g \sqrt{N}},
\]
(20)

where \(g\) is the Lande g-factor.

In a similar manner, it can be shown that the uncertainty in the linewidth is

\[
\sigma_T = \frac{\Gamma}{S \epsilon \sqrt{2N}}.
\]
(21)

And the uncertainty in the value of \(I_0\) is

\[
\sigma_{I_0} = \frac{\sqrt{N}}{\epsilon_p \Gamma \sqrt{\pi}}.
\]
(22)

The fractional error in the intensity is

\[
\frac{\sigma_{I_0}}{I_0} = \frac{1}{\epsilon_p \sqrt{N}}.
\]
(23)

Measurements of Stokes \(Q\) and \(U\) can be used to determine the plane-of-sky azimuth of the magnetic field, \(\phi\), using the equation

\[
\phi = \frac{1}{2} \tan^{-1} \frac{U}{Q}.
\]
(24)

Applying error propagation to this equation gives

\[
\sigma_{\phi}^2 = \left(\frac{\partial \phi}{\partial U}\right)^2 \sigma_U^2 + \left(\frac{\partial \phi}{\partial Q}\right)^2 \sigma_Q^2.
\]
(25)

Assuming that Stokes \(Q\) and \(U\) are integrated over the emission line and that they have equal polarimetric efficiency, their uncertainty is equal to the uncertainty in the intensity divided by the \(Q\) and \(U\) modulation efficiency, \(\epsilon_{Q,U}\). Then

\[
\sigma_{\phi} = \frac{1}{\epsilon_{Q,U} 2p \sqrt{N}},
\]
(26)

where

\[
p = \frac{\sqrt{Q^2 + U^2}}{I}
\]
(27)

is the fraction of linearly polarized light.
Effect of the Background on the Errors

M. Penn et al. (2004) have given a thorough discussion of the effect of background on coronal measurements. For completeness, their derivation is repeated here.

A measurement of the corona which includes a background requires another measurement of the background to subtract from it to obtain the coronal intensity. The measurement $M$ is then given by

$$M = (I + B) - B,$$  \hspace{1cm} (28)

and the corresponding uncertainty in the measurement is

$$\sigma_M^2 = \sigma_I^2 + 2\sigma_B^2$$  \hspace{1cm} (29)

which can be rearranged to be

$$\sigma_M = \sigma_I \left( 1 + 2 \frac{\sigma_B^2}{\sigma_I^2} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (30)

Assuming photon noise, the number of photons in the corona and background can be substituted for the variances resulting in

$$\sigma_M = \sigma_I \left( 1 + 2 \frac{B}{N} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (31)

where $B$ is the number of photons in the background and $N$ is the number of photons in the corona. Then, the presence of a background imposes a penalty in the measurement of the intensity which is given by the factor

$$f = \left( 1 + 2 \frac{B}{N} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (32)

All of the uncertainties in the magnetic field parameters derived above will need to be multiplied by this factor.

Penn et al. also argue that where many samples of the background are taken, as in a spectrograph, then the additional measurement of the background is not necessary and the multiplicative factor becomes

$$f = \left( 1 + \frac{B}{N} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (33)

In both cases, for background dominated measurements, the errors in the magnetic field parameters will scale as the square root of the background level.
Application to the FeXIII 1074.7 nm

Using the value of the line width for the 1074.7 nm FeXIII line of 0.1095 nm and a Lange-g value of 1.5, we can use Equation 17 to find

\[ \sigma_{\lambda_0} = \frac{77.4}{\varepsilon_I \sqrt{N}} \text{ (pm)}. \] (34)

The velocity error from Equation 18 is

\[ \sigma_v = \frac{21.6}{\varepsilon_I \sqrt{N}} \text{ (km/s)}. \] (35)

The uncertainty in the shift due to magnetic field can be derived using equation 20, \( g=1.5, \lambda=1074.7 \text{nm} \) and \( \Gamma=0.1095 \text{nm} \). Then

\[ \sigma_B = \frac{9.6}{\varepsilon_V \sqrt{2N}} \text{ (kG)}. \] (36)

The measured flux in the continuum at the center of the solar disk at a wavelength of 1.1\( \mu \) is \( 0.99 \cdot 10^6 \text{ erg sr}^{-1} \text{ cm}^{-2} \text{ Å}^{-1} \text{ s}^{-1} \) (Allen, 1983), combined with \( 1.81 \cdot 10^{-12} \text{ erg/photon} @ 1.1\mu, 4.25 \cdot 10^{10} \text{ arcsec}^2/sr \) gives \( 1.29 \cdot 10^7 \text{ photons arcsec}^{-2} \text{ cm}^{-2} \text{ Å}^{-1} \text{ s}^{-1} \) at solar disk center.

Then, the number of detected photons will be

\[ N = 1.01 \cdot 10^6 I \Delta \lambda \Delta t \varepsilon D^2 \text{ (photons) }. \] (37)

where \( I \) is the brightness of the corona in units of parts per million (ppm) of the solar disk center intensity, \( \Delta \lambda \) is the wavelength interval in nm, \( \Delta x \) is the spatial size of a resolution element in arcseconds, \( \Delta t \) is the integration time in seconds, \( \varepsilon \) is the system efficiency or throughput (including atmospheric transmission), and \( D \) is the telescope diameter in meters.

We can now derive the errors in the magnetic field parameters by substituting the number of photons into the above equations for the errors including the multiplicative factors for the effect of background. We assume the worst case, where the noise in the measurement of the background is not negligible and the factor of 2 appears in the expression for the effect of the background.

The following are plots of the error in the LOS magnetic field, the LOS velocity, and the POS azimuth of the magnetic field as a function of the telescope aperture, for values of the coronal intensity of 2, 4 and 10 ppm, and for a background level of 5ppm. The coronal intensity in the Fe XIII line is about 10-20 ppm in typical loops and on occasion can reach a brightness of 50 ppm. A value for the background of 5ppm is achieved regularly at excellent coronal sites, while a value of the background of 25ppm corresponds to poor coronal conditions. All plots assume a resolution element of 2x2 arcseconds (\( \Delta x=2 \)), an integration time of 15 minutes (\( \Delta t=900 \)), a spectral range of 0.14 nm corresponding to the filter FWHM and a system efficiency of 5% (\( \varepsilon=0.05 \)). Also, an
ideal polarimeter is assumed with efficiencies of 1 for Stokes I and $1/\sqrt{3}$ for Q, U and V. The POS azimuth plots assume a degree of linear polarization of 5%.

Figure 1 shows the expected error in the LOS magnetic field as a function of the telescope diameter. An error less than 1G is achieved for coronal intensity greater than 10 ppm for apertures greater than 1.5 m in 15 minutes. The plot illustrates the difficulty in measuring the magnetic field for faint coronal features in the presence of significant background levels. Figure 2 shows the LOS velocity as a function of telescope diameter, and Figure 3 shows the POS magnetic azimuth.

![Magnetic Field Error](image)

Figure 1. Error on the LOS magnetic field as a function of telescope diameter for coronal intensity of 2, 4 and 10 ppm and the assumptions given in the text. This calculation does not take into account dual beam observations.
Figure 2. Same as Figure 1, except for LOS velocity.

Figure 3. Same as Figure 1, except for POS azimuth of the magnetic field.
Monte Carlo Simulations

The next section aims to answer the following questions:

- Do the equations derived here provide realistic estimates of the errors on the coronal parameters?
- Can they be applied to both spectrograph type instruments and tunable filter instruments?

To answer these questions, we generated a series of Monte Carlo simulations. The simulations were computed with the following steps.

1. Generate theoretical Stokes profiles

Stokes profiles were computed on a wavelength scale with a spectral resolution of 0.01 nm over a range of ±1.5 nm around 1074.7 nm with the equations for the Stokes profiles

\[ I = I_0 e^{-\frac{(\lambda - \lambda_0)^2}{\Gamma^2}} + B, \quad Q = I p \cos \phi, \quad U = I p \sin \phi, \quad V = -8.09 \cdot 10^{-6} B_{\text{LOS}} \frac{\partial I}{\partial \lambda}, \]

which contain the seven free parameters, \(I_0\), \(\Gamma\), \(\lambda_0\), \(B\), \(p\), \(\phi\) and \(B_{\text{LOS}}\). Each simulation consisted of 100,000 realizations of Stokes profiles generated with seed values generated with

- \(I_0\) varied randomly with uniform distribution between 1 and 100 ppm, for simplicity only integer \(I_0\) values were used
- \(\Gamma\) set to 0.1095 nm,
- \(\lambda_0\) varied randomly around 1074.617 nm with uniform distribution between ±10 km/s,
- \(B\) set to 5 ppm,
- \(p\) set to 0.05,
- \(\phi\) varied randomly with uniform distribution between ±90 degrees, and
- \(B_{\text{LOS}}\) varied randomly with uniform distribution between ±50 G.

The assumed telescope parameters were 1.5m aperture, 2 arcseconds spatial resolution, 900 seconds total integration time and dual beam observations. The system throughput was set to 0.0943 at 1074.7 nm based on the COSMO LC flux budget (Oakley et al., 2015).

2. Observe the Stokes profiles

For the spectrograph mode, the Stokes profiles were sampled at the 0.01 nm resolution of the seed profiles and the photon levels for each spectral element were computed using the total integration time of 900 s. For the Filtergraph mode, the Stokes profiles were sampled by COSMO Filtergraph transmission profiles appropriate to the 5 stage Lyot filter with a FWHM of 0.14 nm. The measurements were made sequentially and the integration time was distributed among the different filter tunings.

Noise was added to the observations of each Stokes parameter assuming the observations are limited by photon noise and taking into account the modulation efficiency of the polarimeter according to the formulation by del Toro Iniesta and Collados (2000).
noise was randomly generated with a Gaussian distribution with a standard deviation given by $\sqrt{N}/\varepsilon_S$, where $N$ is the number of photons observed in a wavelength bin for the spectrograph, or through a filter for the Filtergraph, and $\varepsilon_S$ is the modulation efficiency for the Stokes state. The COSMO filtergraph will employ a beamsplitter and observe with two beams simultaneously, effectively doubling the number of collected photons. The dual beam mode was accounted for by dividing the resulting noise estimates and theoretical predictions by $\sqrt{2}$.

3. Fit the Stokes profiles

Next, the observations were fit using Powell minimization. The Stokes profiles of the model observations were generated as in Step 1 above and observed as in Step 2. The $\chi^2$ used for minimization was

$$
\chi^2 = \frac{1}{(4n_\lambda - 7)} \sum_{\lambda} \sum_{S=I,Q,U,V} [N_{obs}(S,\lambda) - N_{model}(S,\lambda)]^2 \frac{\varepsilon_S^2}{N(I,\lambda)}.
$$

$N$ is the number of photons for a Stokes parameter and wavelength and $n_\lambda$ is the number of wavelength points or filter tunings.

For the spectrograph simulations, the initial guess was set to the correct value. For the filtergraph experiments, the initial guess for the parameters was generated from the combinations of the observations using algorithms like the magnetograph formula. Comparison of simulations using the correct value and estimated values showed no difference. The routine typically required 5-10 iterations to converge.

Results for Spectrograph

The results for the spectrograph simulation are shown in Figure 4. The solid dots show the errors which are the standard deviations of the differences between the seed parameters and fit parameters evaluated for each 1 ppm coronal emission bin. The modulation efficiencies used correspond to the nominal 2-element COSMO LC modulator and have efficiencies of 0.904, 0.530, 0.530 and 0.613 for Stokes I, Q, U and V at 1074 nm.

The result for LOS magnetic field strength is shown in panel a). The cross indicates the 1 G requirement at 10 ppm. The dotted line shows the variation with emission that scales as $\frac{1}{\sqrt{N}} \left(1 + 2\frac{B}{N}\right)^{\frac{1}{2}}$ and goes through the requirement. The computed errors on the field strength are a factor of 0.44 of the dotted requirement line. The solid line is the theoretical prediction using Equation 36 with a Stokes I efficiency of 0.904 and the emission scaling as $\left(1 + \frac{B}{N}\right)^{\frac{1}{2}}$. The dashed line is the theoretical prediction using Equation 36 and the emission scaling as $\left(1 + 2\frac{B}{N}\right)^{\frac{1}{2}}$. Clearly, the prediction using the factor of 2 in the penalty factor term fits the data better, despite the expectation that the 2 should be replaced by a 1 in the case of spectrograph measurements when a large number of background measurements are taken.
Figure 4. Errors on the parameters as a function of the coronal line emission strength for the spectrograph simulation are shown as the solid dots. The solid lines in all the plots show the theoretical predictions using the analytic expressions multiplied by the penalty factor with the value of 2. a) Field strength error. The cross indicates the 1 G requirement at 10 ppm. The dotted line shows the variation with emission that scales as \((1 + 2 \frac{B}{N})^{1/2}\) that goes through the requirement. The dashed line is the theoretical prediction using the penalty factor without the factor of 2.
Figure 5. Correlations of the errors on the parameters. The Pearson correlation coefficients are shown. The line width shows a significant correlation with the central intensity and the background level.
The Doppler velocity errors are shown in panel b). There the cross shows the 30 m/s requirement in 30 seconds scaled to 5.5 m/s for the 900 second integration time of the simulation. The solid line is Equation 35 scaled by the penalty factor with the value of 2.

The other solid curves show the comparison with theory using the penalty factor with the value of 2. In general, the agreement with the analytic theory is excellent. The one exception is the prediction of the error on the line width which exceeds the prediction by about a factor of 2. Even at the elevated level of error, the line width is still recovered to about 1 m/s at 10 ppm emission.

To investigate the reason for the disagreement, correlations between the errors were computed and are shown in Figure 5 along with the linear Pearson correlation coefficients. Figure 5 shows that the error on the line width is highly correlated with the errors on the central intensity and background level. This violates the assumption of uncorrelated errors in the derivations and most likely accounts for the errors on the line width exceeding the predictions.

The simulation shows that spectrograph measurements can exceed the COSMO requirement on the field strength by a factor > 2. This assumes that the spectrograph will simultaneously observe the full COSMO field of-view of 1 degree at spatial resolution of 2 arcseconds. This is not feasible with current technologies.

Results for the Filtergraph

The disadvantage of a filtergraph instrument compared to a spectrograph is that the filtergrams are taken sequentially. The advantage is that they are taken over the entire field of-view. If the number of filtergraph tunings can be made smaller than the number of required slit positions, then the filtergraph will have an advantage. For the COSMO filtergraph, the 900 second integration time needs to be allocated to the multiple filter tunings. Several simulations were run with variations of the wavelengths of the filter and the time allocated to each and different modulation schemes.

1. Filtergraph simulation 1

The first filter simulation attempted to mimic a spectrograph with 14 filter tunings as shown in Figure 6. One background measurement is taken 0.8 nm away from line center, and 13 tunings are taken around line center with a spacing of 0.03 nm. The nominal emission line profile is shown in Figure 6 b). This simulation was computed with the 900 s integration time equally divided among the 14 filter tunings. The modulation efficiencies are the same as in the spectrograph simulation. The results for the errors are shown in Figure 8. The simulated errors are a factor of 1.08 above the requirement for the field strength.

Note that the theoretical expressions shown as the solid lines of Figure 8 used the actual number of photons observed through the filters, not the integrated number of photons in the emission line as in the spectrograph simulation. Since the filter measurements were able to observe only 0.34 of the total photons in the emission line, the theoretical errors for the filter measurements are correspondingly higher than the spectrograph errors. In addition, the simulated errors are about a factor of 1.6 higher than the predictions for the field strength, Doppler velocity and central intensity. The line width is a factor of 3.81 above prediction but the errors on the degree of polarization and field azimuth are
consistent with theory. The large discrepancy in the observed and predicted errors on the line width are due to the correlations of the line width error and intensity and background errors which are shown for this simulation in Figure 7.

![Figure 6](image1.png)

Figure 6. a) 14 filter tunings for the filtergraph simulation 1. b) The nominal emission line profile.

![Figure 7](image2.png)

Figure 7. Correlation of the line width errors with the intensity and background errors for filtergraph simulation 1.
Figure 8. Errors on the parameters as a function of the emission strength for filtergraph simulation 1 are shown as the solid dots. The solid lines in all the plots show the theoretical prediction using the analytic expressions with the number of photons observed through the filter multiplied by the penalty factor with the value of 2. a) Field strength errors. The cross indicates the 1 G requirement at 10 ppm. The dotted line shows the variation with emission that scales as \( \left( 1 + \frac{B}{N} \right)^{\frac{1}{2}} \) that goes through the requirement.
2. Filtergraph simulation 2

Another advantage of filtergraph measurements is that different amounts of time can be spent at different wavelengths. This offers the possibility of minimizing the errors on selected parameters. This begs the question: What is the optimal placement of filter tunings and optimal allocation of integration time to minimize the errors on a given parameter? We believe that the optimal weights are simply given by Equation 7. Combining Equations 7 and 9 we can obtain

\[
W_i = \frac{\left( \frac{\partial I}{\partial \lambda_i} \right)^2}{\sigma_{\lambda i}^2}.
\]  

(38)

For the magnetic field and velocity measurements, the parameter of interest is \(\lambda_o\). In that case the optimal weight assuming photon noise, will be given by the derivative of the intensity with respect to wavelength squared divided by the intensity. To determine the optimal measurement strategy, one needs to use the filter transmission profiles as a set of basis functions and create a superposition of the transmission profiles weighted by the observing time that approximates the optimal weighting function.

Figure 9 shows the optimal weighting function for magnetic and velocity signals along with the combination of filters that approximates it. Some weight at line center was purposely left in so that measurements of the intensity, line width, linear polarization and azimuth will not be unduly compromised and some weight away from line center was included to help with the determination of the background.

![Figure 9. Optimal weighting function (solid line) and the superposition of filter functions that approximates it (dashed line).](image-url)
Figure 10. Errors on the parameters as a function of the emission strength for filtergraph simulation 2 are shown as the solid dots. The solid lines in all the plots show the theoretical prediction using the analytic expressions with the number of photons observed through the filter multiplied by the penalty factor with the value of 2. All other symbols and curves are as previously described.
The filter tunings and integration times used to achieve the weighting of Figure 9 are

<table>
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<th>λ (nm)</th>
<th>-0.80</th>
<th>-0.16</th>
<th>-0.12</th>
<th>-0.08</th>
<th>0.0</th>
<th>0.08</th>
<th>0.12</th>
<th>0.16</th>
</tr>
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<tr>
<td>Time (s)</td>
<td>50</td>
<td>100</td>
<td>250</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>250</td>
<td>100</td>
</tr>
</tbody>
</table>

The modulation efficiencies for simulation 2 are the same as that for simulation 1. The errors for filtergraph simulation 2 are shown in Figure 10. The errors on the magnetic field strength are reduced from 1.08 of the requirement for simulation 1 to 0.92 of the requirement for simulation 2. The errors on the magnetic field and velocity more closely approach the predicted values than in simulation 1 but remain about 25% above the predicted values. Errors on the azimuth and linear polarization are unchanged, and the errors on the intensity and line width have been increased slightly.

1. Filtergraph simulation 3

Another dimension of optimization is possible by modifying the modulator design to add efficiency to one of the Stokes parameters at the expense of the others. To investigate this, the COSMO modulator was re-optimized to increase the Stokes V efficiency. The resulting modulator has efficiencies of 0.834, 0.482, 0.482 and 0.659 for Stokes I, Q, U and V at 1074 nm. Increasing the Stokes V efficiency has decreased the efficiency of the other 3 Stokes parameters. This is a small increase in the Stokes V efficiency of 8% over the previous case. The results of the simulation shown in Figure 11 show a decrease in the errors on the magnetic field which are now 0.85 of requirement. The 8% increase in efficiency for Stokes V has resulted in a direct decrease of the noise by the same factor consistent with the theory.

![Figure 11](image-url)  

Figure 11. Errors on the magnetic field and velocity for filtergraph simulation 3. The errors on the magnetic field have been reduced proportionally to the increase in modulation efficiency of Stokes V.
Conclusions

The COSMO Large Coronagraph is required to achieve a 1 G error level in the LOS magnetic field in 15 minutes with 2 arcseconds spatial resolution. This will require an aperture of at least 1.5 meters and the LC must have very low levels of background light. This demands that the site be of exceptional coronal quality, and that the coronagraph be designed to insure a very low level of instrumental scattered light.

The analytic expressions for the errors on the coronal emission line parameters derived here are verified to be reasonably reliable predictors of the errors so long as one uses the total number of photons in the emission line for the spectrograph, and the total number of photons observed through the filters for the filtergraph mode. One notable exception is the errors on the line width that are a factor of 2 above the predicted levels. This is probably due to correlations of the errors on the intensity and background with the line width errors. The noise appears to scale the same way as a function of background level for a spectrograph and filtergraph in disagreement with expectations. The errors on the linear polarization and magnetic azimuth appear to be very well behaved and agree with the theory in all simulations run so far. This may be due to an isolation of the linear polarization parameters from the other ones.

The simulations show that for equally spaced filter observations using equal observing times for the filters the errors are 1.08 time the COSMO requirement for the magnetic field and significantly below the requirement on the velocity. Reduction of the errors through optimization is possible by two avenues, optimizing the measurements in terms of wavelength position and time spent at each wavelength, and reducing the noise on the magnetic field through increasing the Stokes V modulation efficiency. The modest optimizations presented here were sufficient to reduce the errors on the magnetic field to be below the requirement by 15%. Much larger reductions may be possible and will be investigated in future work.

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References


